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Turbulent Mass Transfer in Jet Flow and Bubble Flow: A Reappraisal of Levich's Theory

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In a recent paper, Davies (1972*a*) used a modified Levich theory to obtain a prediction of the rate of gas absorption by turbulent water jets (Ting, 1967; Davies and Ting, 1967) and kerosene jets (Hameed, 1969; Davies and Hameed, 1971). Davies' predictive equation for the mass transfer coefficient was

$$k_L = 0.32(D\,\rho\,V_O{}^3/\sigma)^{1/2} \qquad (1)$$

where V_O is the characteristic turbulence velocity. Although Davies (1972*a*) mentioned that the liquid turbulence decays appreciably along the free jet, he neglected this effect in the mass transfer calculations and took V_O to be equal to the shear velocity in the tube:

$$V_O = V_* = 0.2\overline{V}\,Re^{-1/8} \qquad (2)$$

The mass transfer, then, was assumed to be governed by the developed turbulence within the tube. If this were a good assumption, Equations (1) and (2) would also be expected to provide a prediction of the gas absorption rate by discrete bubbles transported in turbulent pipe flow of water (Lamont, 1966; Lamont and Scott, 1966, 1970). Here the turbulence within the tube is controlling the mass transfer process. However, this expectation is not met. In Table 1, the jet flow and bubble flow data are interpreted by an equation of the form

$$k_L = C_L(D\,\rho\,V_O{}^3/\sigma)^{1/2} \qquad (3)$$

where V_O is calculated from Equation (2), and C_L is the constant which gives the best agreement (mean-square criterion) between the measured and predicted mass transfer coefficients for a given physical system. It is seen that Equation (1) underestimates the bubble flow data by a factor of 10. In addition, while Equation (1) is in excellent agreement with the water-jet data, it overestimates the kerosene-jet data by about 50%.

Davies and Hameed (1971) were unable to explain the low C_L value for their kerosene jets and suggested that the absolute values of the proportionality constants in the Levich theory need reexamining. Also, Coeuret and Ronco (1973) could not explain the C_L variation between the water-jet and kerosene-jet results. They concluded that the Levich theory interpretation of the turbulent mass transfer process is not yet sufficient. The present paper investigates the possibility that the C_L variation in the jet flow case might be due to the incorrect specification of V_O (due to the neglect of turbulence decay) rather than to an implied limitation in the Levich theory. The motivation, then, is to allow an assessment of the Levich theory on a more realistic and uniform basis. It is shown that if the effects of turbulence decay are included for the liquid jets, a new and more invariant value of the Levich constant C_L results.

REPRESENTATION OF TURBULENCE DECAY IN LIQUID JETS

The bulk turbulence properties of the free jet, which are initially characteristic of the nearly isotropic conditions in the core of the nozzle, are assumed to follow the decay laws describing the turbulent flow downstream of a grid (Batchelor and Townsend, 1948; Batchelor, 1967). For a region downstream of the nozzle, the turbulence energy is taken to decay according to the law of the initial period, while for larger distances downstream, the law of the final period is applied. The short transition region which separates these two periods of decay will be neglected. The length of duration of the initial period t^* is of the order of a few characteristic eddy times during the early stages of decay (Batchelor, 1967). Since the experimental data of Batchelor and Townsend (1948) indicate that $t^* \sim 10T$, it is reasonable to take $t^* = 10L(x = 0)/V(x = 0)$ for jet flow.

The initial period of decay is characterized by $V^{-2} = C_1(t - t_1)$ and $\lambda^2 = 10\nu(t - t_1)$, where $t = x/\overline{V}$ and $\lambda^2 = 10\nu\,L/V$, and the constants C_1 and t_1 are determined by requiring continuity of L and V at $x = 0$. The results are

$$V = V(x=0)\left\{1 + \frac{V(x=0)}{L(x=0)}\,t\right\}^{-1/2}$$

TABLE 1. MASS TRANSFER DATA FOR JET FLOW AND BUBBLE FLOW AS DESCRIBED BY EQUATION (3)

Jet flow		Discrete bubble flow in pipes
Ting, 1967; Davies and Ting, 1967	Hameed, 1969; Davies and Hameed, 1971	Lamont, 1966; Lamont and Scott, 1966, 1970
Liquid: water	Liquid: kerosene	Liquid: water
Gas: CO_2, H_2	Gas: CO_2, H_2, He	Gas: CO_2
$C_L = 0.32$	$C_L = 0.20$	$C_L = 3.2$
$E = 15\%$	$E = 24\%$	$E = 48\%$
$S = 0.81$	$S = 0.67$	$S = 1.7$

C_L is the constant minimizing the sum of the squares of the deviations between the measured and predicted mass transfer coefficients.
E is the average percentage error between the measured and predicted mass transfer coefficients.
S is the slope of the least-squares line through the points on a plot of the predicted vs. measured mass transfer coefficient.

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Table 2. Mass Transfer Data for Jet Flow as Described by Equation (7)

Jet flow	
Ting, 1967; Davies and Ting, 1967	Hameed, 1969; Davies and Hameed, 1971
Liquid: water	Liquid: kerosene
Gas: CO_2, H_2	Gas: CO_2, H_2, He
$C_L' = 3.0$	$C_L' = 2.6$
$E = 17\%$	$E = 28\%$
$S = 1.0$	$S = 0.72$

$$\text{and} \quad L = L(x=0)\left\{1 + \frac{V(x=0)}{L(x=0)}t\right\}^{1/2} \qquad (4)$$

where $L(x = 0) \simeq 0.03d$ for the range of Reynolds numbers used (Martin and Johanson, 1965), and $V(x=0) \simeq V_* = V_O$ (Hinze, 1959). The length of the initial period ranges from 0.45 to 0.85 cm for the experimental conditions employed by Ting and Hameed.

The final period of decay is characterized by $V^{-2} = C_2(t - t_2)^{5/2}$, $\lambda_2 = 4\nu(t - t_2)$, and $\lambda = \sqrt{2/\pi}\, L$, where the constants C_2 and t_2 are determined by requiring continuity of L and V for $t = t^*$. The results are

$$V = V(t^*)L(t^*)^{5/2}[L(t^*)^2 + 2\pi\nu(t - t^*)]^{-5/4}$$

and

$$L = [L(t^*)^2 + 2\pi\nu(t - t^*)]^{1/2} \quad \text{for} \quad t > t^* \qquad (5)$$

MASS TRANSFER PREDICTIONS

Since $V(x = 0) \simeq V_O$, it is reasonable to take $V_O = V(t)$ for the mass transfer calculation over the length of the jet. The local mass transfer coefficient, then, may be represented by the Levich theory as

$$k_L(t) = C_L'[D\rho V(t)^3/\sigma]^{1/2} \qquad (6)$$

and the overall mass transfer coefficient is

$$k_L = C_L'\left(\frac{D\rho}{\sigma}\right)^{1/2}\left[\frac{1}{t_{\text{exp}}}\int_0^{t_{\text{exp}}} V(t)^{3/2}\,dt\right]$$

$$\equiv C_L'\left(\frac{D\rho}{\sigma}\right)^{1/2}\overline{V}_O^{3/2} \qquad (7)$$

where $t_{\text{exp}} = L_S/\overline{V}$, and $V(t)$ is obtained from Equations (4) and (5). After the integration is performed, the effective characteristic eddy velocity $\overline{V}_O$ is found to be

$$\overline{V}_O = \frac{1}{t_{\text{exp}}^{2/3}}\left\{4L(x=0)V(x=0)^{1/2}\left[\left(1 + \frac{V(x=0)t^*}{L(x=0)}\right)^{1/4} - 1\right] + \frac{4V(t^*)^{3/2}L(t^*)^{15/4}}{7\pi\nu}[L(t^*)^{-7/4} - [L(t^*)^2 + 2\pi\nu(t_{\text{exp}} - t^*)]^{-7/8}]\right\}^{2/3} \qquad (8)$$

The mass transfer results for the turbulent jets, as described by Equation (7), are summarized in Table 2. It is seen that the C_L' values for the water and kerosene data (3.0 vs. 2.6) are in better agreement than the corresponding C_L values (0.32 vs. 0.20), that is, 15 and 60% variation, respectively. In addition, these C_L' values are close to the value of 3.2 deduced from the bubble flow data (Table 1). The k_L values calculated by Equation (7) are insensitive to t^*. For example, if it is assumed that $t^* = 5L(x = 0)/V(x = 0)$, the calculated C_L' values for the water jets and kerosene jets are 3.2 and 3.1, respectively. Even if the turbulence energy is taken to decay according to the law of the initial period over the entire jet length, the calculated C_L values for the water jets and kerosene jets are 3.1 and 2.2, respectively. Levich's (1962) original predictive equation had the same form as Equation (1) but with a proportionality constant of 1 instead of 0.32. Levich's suggested constant is in much better agreement with the values of C_L' calculated in this paper. The sensitivity parameter S given in Tables 1 and 2 provides a measure of the sensitivity of the mass transfer models to the parameters governing the mass transfer process. The calculations for liquid-jet gas ab-

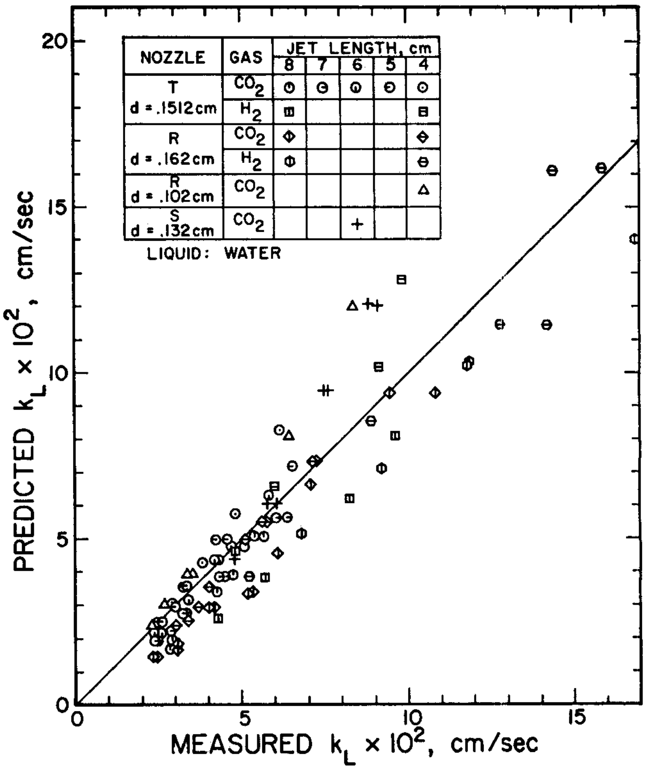

Fig. 1. Comparison of predicted with measured overall mass transfer coefficient for the data of Ting (1967).

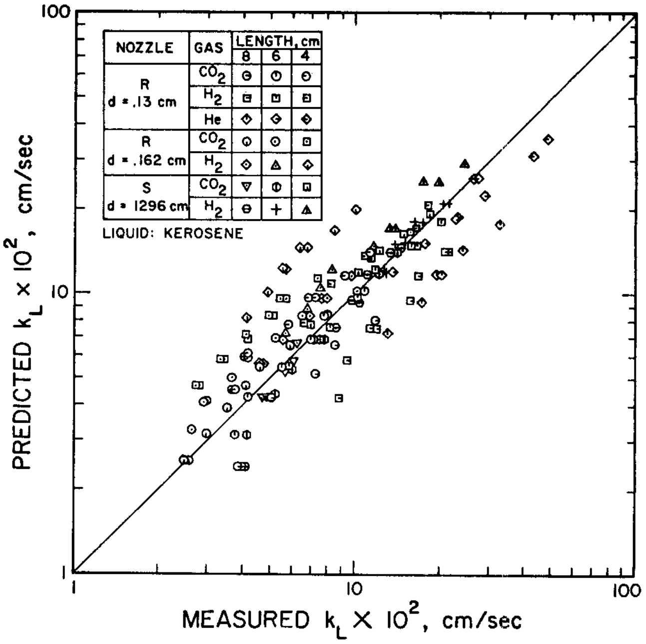

Fig. 2. Comparison of predicted with measured overall mass transfer coefficient for the data of Hameed (1969).

sorption based on Equation (7) yield better values for S than those based on Equations (2) and (3), which neglect turbulence decay. Figures 1 and 2 compare the predicted values of the overall mass transfer coefficient with the measured values for the water and kerosene jets. When we consider the wide variety of experimental conditions covered, the scatter does not appear to be excessive. However, careful examination reveals a definite stratification with experimental parameters (nozzle size, jet length, gas used, etc.) This stratification is considered to be due to limitations in Levich's mass transfer model. Also, the high sensitvity and error shown in Table 1 for the bubble flow case raises questions as to the applicability of the Levich theory. More complete discussion of mass transfer models has recently been provided by Brumfield et al. (1975) and Theofanous et al. (1976).

DISCUSSION

Davies and Hameed (1971) assumed that the local eddy size was equal to the measured size l of the protuberances on the jet surface, and, on this basis, they associated the rate of turbulence damping with the rate of growth of l along the jet. They concluded that the damping of turbulence by surface tension forces is about twenty-seven times greater than that by bulk viscous forces. Our approach, on the other hand, considers only viscous damping, totally ignoring the role of surface tension. The following points may be made in considering the relative merits of each approach:

1. In light of the magnitude of l, it appears physically unrealistic that the length scale of turbulence (macroscale) could be as large as l. The observed magnitude of the protuberances l was up to the order of $2d$ (Davies, 1972*b*). On the other hand, the macroscale calculated with Equation (5) reaches, for the experimental conditions employed by Ting and by Hameed, a maximum value of about $0.3d$.

2. According to Davies, the characteristic eddy velocity V_O may be related to the size of surface protuberances l by

$$\rho V_O^2 \simeq \frac{4\sigma}{l} \tag{9}$$

Over the length of the free jet, l increases by about a factor of 10 (Davies, 1972*b*), so that according to Equation (9), V_O decreases by a factor of 3. On the other hand, calculations based on a viscous damping mechanism [Equations (4) and (5)] show (for the same conditions) that V_O is reduced by a factor of 25. In other words, viscosity gives one order of magnitude greater damping than surface tension.

3. The kinematic viscosity of kerosene is approximately twice that of water, while its surface tension is only one half that of water. For a viscous damping mechanism, therefore, kerosene would exhibit faster decay than water, while the opposite would be true for a surface tension damping mechanism. A simple way to see that the mass transfer data, in fact, support a viscous decay mechanism will now be described. From the results of Tables 1 and 2 we have

$$\overline{V}_{Ow} = \left(\frac{0.32}{3}\right)^{2/3} V_{Ow} = 0.23\, V_{Ow} \tag{10}$$

and

$$\overline{V}_{Ok} = \left(\frac{0.2}{2.6}\right)^{2/3} V_{Ok} = 0.18\, V_{Ok} \tag{11}$$

That is, a 30% faster decay (in some average sense) for kerosene compared to that of water is implied by the mass transfer data.

CONCLUSIONS

For time (space) dependent turbulent flows, the variation of turbulence properties must be accounted for in calculating local and average mass transfer coefficients. An example is given for absorption in turbulent liquid jets. A consistent behavior of the Levich theory for mass transfer emerges from this treatment. Further, a new and more invariant value of the Levich constant results. This value is also consistent with mass transfer data from bubble flow, and it is of order one as originally suggested by Levich.

ACKNOWLEDGMENT

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NOTATION

C_L = Levich constant based on constant V_O, Equation (3)
C_L' = Levich constant based on turbulence decay, Equation (6)
d = pipe diameter
D = molecular diffusivity
E = average percentage error
k_L = mass transfer coefficient
l = size of protuberance on jet surface
L = macroscale of turbulence
L_s = jet length
Re = Reynolds number, $\overline{V}d/\nu$
S = sensitivity parameter
t = time
$t_{\exp}$ = exposure time for mass transfer
t^* = time of duration of the initial period
T = characteristic eddy time, L/V
V = turbulence intensity
$\overline{V}$ = bulk velocity
V_O = Levich's characteristic turbulence velocity
$\overline{V}_O$ = effective characteristic eddy velocity, Equation (8)
V_* = shear velocity
x = spatial coordinate along jet axis

Greek Letters

λ = microscale of turbulence
μ = molecular viscosity
ν = kinematic viscosity, μ/ρ
ρ = density
σ = surface tension

Subscripts

k = kerosene
w = water

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An Experimental Study of Diffusion From a Line Source in a Turbulent Boundary Layer

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This paper summarizes part of an experimental investigation (Collins, 1973) concerning slot injection of Poly(ethylene oxide) solution concentrate into a two-dimentional developing turbulent boundary layer in channel flow. Experimentally measured velocity and concentration profiles were used to evaluate the effect of injection concentration and rate on the concentration boundary layers. These profiles were used to evaluate wall concentration and diffusing plume thickness for injections of pure water and various concentrations of Poly(ethylene oxide). The data are the only experimental results for turbulent diffusion measurements close to the source.

DIFFUSION IN A TURBULENT BOUNDARY LAYER

An experimental study of diffusion of ammonia from a slot into a turbulent air boundary layer serves as the basis for characterizing turbulent diffusion. Poreh and Cermak (1964) found a series of four stages or zones in the development of a concentration boundary layer. They defined a relative rate parameter β to differentiate between zones:

$$L_\delta = \delta \Big/ \frac{d\delta}{dx} \tag{1}$$

$$L_\lambda = \lambda \Big/ \frac{d\lambda}{dx} \tag{2}$$

$$\beta = L_\lambda / L_\delta \tag{3}$$

Initial Zone: In this region closest to the source, the concentration boundary layer grows slowly by molecular diffusion through the laminar sublayer. The characteristic length λ is the same order of magnitude as the laminar sublayer.

Intermediate Zone: In this zone, there is very rapid growth due to high turbulence intensity near the wall. The diffusing plume is submerged in the boundary layer and is considerably larger than the laminar sublayer. Axial concentration gradients are much smaller than vertical gradients, and the ratio β is small. The mean concentration profiles can be described by the following dimensionless curve, proposed by Morkovin (1965):

$$\frac{C}{C_w} = \exp\left[-0.693(y/\lambda)^{1.5}\right] \tag{4}$$

The intermediate zone lies in the range $0.15 < \lambda/\delta < 0.36$ and $0.08 < \beta < 0.38$.

Transition Zone: In this region, the plume growth rate is slowed because of less turbulence in the outer region of the boundary layer. The profile begins to gradually change shape. This region begins approximately 18 boundary-layer thickness from the source and ends at 60. At the end of this zone β increases to unity, and the value of λ/δ approaches 0.64.

The Final Zone: In the final zone, the concentration boundary layer coincides with the shear layer. The value of λ/δ remains constant (0.64) and is expressed as

$$\frac{C}{C_w} = \exp\left[-0.693(y/\lambda)^{2.15}\right] \tag{5}$$

EXPERIMENT

The investigation was conducted in the water tunnel of the School of Chemical Engineering, Georgia Institute of Technology. The apparatus was a single-pass, continuous flow, low-speed water tunnel consisting of an inlet section, a test section, and an exit duct, all constructed of 8 mm aluminum plate. The system was operated at the maximum capacity of 900 l of water/min for all runs. This gave a free stream velocity and boundary-layer thickness of 0.8 m/s and 2 cm, respectively.

Concentration measurements were made on 25 ml samples, which were withdrawn through a Pitot probe. Analyses were determined colormetrically (Bausch and Lomb Spectronic 20) from the intensity of a tracer